

# Quantum many-body dynamics in optomechanical arrays

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We study the nonlinear driven dissipative quantum dynamics of an array of optomechanical systems. At each site of such an array, a localized mechanical mode interacts with a laser-driven cavity mode via radiation pressure, and both photons and phonons can hop between neighboring sites. The competition between coherent interaction and dissipation gives rise to a rich phase diagram characterizing the optical and mechanical many-body states. For weak intercellular coupling, the mechanical motion at different sites is incoherent due to the influence of quantum noise. When increasing the coupling strength, however, we observe a phase transition towards a regime of phase-coherent mechanical oscillations. This transition and the phase diagram of the system are studied using a Gutzwiller ansatz for the dynamics of the driven-dissipative system.

*Introduction.* - Recent experimental progress has brought optomechanical systems into the quantum regime: A single mechanical mode interacting with a laser-driven cavity field has been cooled to the ground state [1, 2]. Several of these setups, in particular optomechanical crystals, offer the potential to be scaled up to form optomechanical arrays. Applications of such arrays for quantum information processing [3, 4] have been proposed lately. Given these developments, one is led to explore quantum many-body effects in optomechanical arrays. In this work, we analyze the nonlinear photon and phonon dynamics in a homogeneous two-dimensional optomechanical array. In contrast to earlier works [3–6], here we study the array’s quantum dynamics beyond a quadratic Hamiltonian. To tackle the non-equilibrium many-body problem of this nonlinear dissipative system, we employ a mean-field approach for the collective dynamics. First, we discuss photon statistics in the array, in particular how the photon blockade effect [7] is altered in the presence of intercellular coupling. The main part of the article focusses on the transition of the collective mechanical motion from an incoherent state (due to quantum noise) to an ordered state with phase-coherent mechanical oscillations. For these dynamics, the dissipative effects induced by the optical modes play a crucial role. On the one hand, they allow the mechanical modes to settle into self-induced oscillations [8–15] once the optomechanical amplification rate exceeds the intrinsic mechanical damping, see Fig. 1(b). On the other hand, the fundamental quantum noise (e.g. cavity shot noise) diffuses the mechanical phases and prevents the mechanical modes from synchronizing. This interplay leads to an elaborate phase diagram characterizing the transition. To gain further insight, we develop a semiclassical model describing the coupling of the mechanical phases and the influence of quantum noise.

While true long-range order is prohibited for a two-dimensional system with continuous symmetry, a Beresinskii-Kosterlitz-Thouless transition towards a state with quasi-long range order is possible. The ordered mechanical phase thus resembles the superfluid phase in

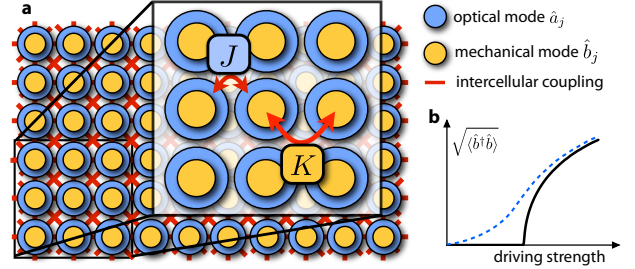


Figure 1. (a) Optomechanical array with localized mechanical ( $\hat{b}_j$ ) and laser-driven optical modes ( $\hat{a}_j$ ) at each site. The optical and mechanical coupling between neighboring sites is set by  $J$  and  $K$ , respectively. (b) Onset of self-induced oscillations for an isolated mechanical mode as a function of laser driving strength (schematic). The classical dynamics (black solid line) show a bifurcation. Quantum fluctuations blur the transition (dashed blue line) and generate a mechanical state whose phase is completely undetermined, see also Fig. 3(b).

two dimensional cold atomic gases [16] or Josephson junction arrays [17]. Notably, optomechanical arrays combine the tunability of optical systems with the robustness and durability of an integrated solid-state device. Other driven dissipative systems that have been studied with regard to phase transitions recently include cold atomic gases [18–23], nonlinear cavity arrays [24, 25] and optical fibres [26]. In a very recent work and along the lines of [18], the preparation of long-range order for photonic modes was proposed using the linear dissipative effects in an optomechanical array [6]. Our work adds the novel aspect of a mechanical phase transition to the studies of driven dissipative many-body systems.

*Model.* - We study the collective quantum dynamics of a two-dimensional homogeneous array of optomechanical cells (Fig. 1). Each of these cells consists of a mechanical mode and a laser driven optical mode that interact via the radiation pressure coupling at a rate  $g_0$  ( $\hbar = 1$ ):

$$\hat{H}_{\text{om},j} = -\Delta \hat{a}_j^\dagger \hat{a}_j + \Omega \hat{b}_j^\dagger \hat{b}_j - g_0 (\hat{b}_j^\dagger + \hat{b}_j) \hat{a}_j^\dagger \hat{a}_j + \alpha_L (\hat{a}_j^\dagger + \hat{a}_j). \quad (1)$$

The mechanical mode ( $\hat{b}_j$ ) is characterized by a frequency  $\Omega$ . The cavity mode ( $\hat{a}_j$ ) is transformed into the frame rotating at the laser frequency ( $\Delta = \omega_{\text{laser}} - \omega_{\text{cav}}$ ) and driven at the rate  $\alpha_L$ . In the most general case, both photons and phonons can tunnel between neighboring sites  $\langle ij \rangle$  at rates  $J/z$  and  $K/z$ , where  $z$  denotes the coordination number. The full Hamiltonian of the array is given by  $\hat{H} = \sum_j \hat{H}_{\text{om},j} + \hat{H}_{\text{int}}$ , with

$$\hat{H}_{\text{int}} = -\frac{J}{z} \sum_{\langle ij \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger) - \frac{K}{z} \sum_{\langle ij \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_i \hat{b}_j^\dagger) \quad (2)$$

To bring this many-body problem into a treatable form, we apply the Gutzwiller ansatz  $\hat{A}_i^\dagger \hat{A}_j \approx \langle \hat{A}_i^\dagger \rangle \hat{A}_j + \hat{A}_i^\dagger \langle \hat{A}_j \rangle - \langle \hat{A}_i^\dagger \rangle \langle \hat{A}_j \rangle$  to Eq. (2). The accuracy of this approximation improves if the number of neighboring sites  $z$  increases. For identical cells, the index  $j$  can be dropped and the Hamiltonian reduces to a sum of independent contributions, each of which is described by

$$\hat{H}_{\text{mf}} = \hat{H}_{\text{om}} - J(\hat{a}^\dagger \langle \hat{a} \rangle + \hat{a} \langle \hat{a}^\dagger \rangle) - K(\hat{b}^\dagger \langle \hat{b} \rangle + \hat{b} \langle \hat{b}^\dagger \rangle). \quad (3)$$

Hence, a Lindblad master equation for the single cell density matrix  $\hat{\rho}$ ,  $d\hat{\rho}/dt = -i[\hat{H}_{\text{mf}}, \hat{\rho}] + \kappa \mathcal{D}[\hat{a}]\hat{\rho} + \Gamma \mathcal{D}[\hat{b}]\hat{\rho}$  can be employed. The Lindblad terms  $\mathcal{D}[\hat{A}]\hat{\rho} = \hat{A}\hat{\rho}\hat{A}^\dagger - \frac{1}{2}\hat{A}^\dagger\hat{A}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{A}^\dagger\hat{A}$  take into account photon decay at a rate  $\kappa$  and mechanical dissipation (here assumed due to a zero temperature bath) at a rate  $\Gamma$ .

*Photon statistics.* - Recently, it was shown that the effect of photon blockade [7] can appear in a single optomechanical cell: The interaction with the mechanical mode induces an effective nonlinearity for the photon field of strength  $g_0^2/\Omega$  [7, 27]. Hence, the presence of a single photon can hinder other photons from entering the cavity. To observe this effect, the nonlinearity must be comparable to the cavity decay rate, i.e.  $g_0^2/\Omega \gtrsim \kappa$ , and the laser drive weak ( $\alpha_L \ll \kappa$ ) [7, 28].

To study nonclassical effects in the photon statistics, we analyze the steady-state photon correlation function  $g^{(2)}(\tau) = \langle \hat{a}^\dagger(t) \hat{a}^\dagger(t+\tau) \hat{a}(t+\tau) \hat{a}(t) \rangle / \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2$  [29] at equal times ( $\tau = 0$ ), with  $g^{(2)}(0) = 1$  for a coherent state, and  $g^{(2)}(0) < 1$  ( $> 1$ ) indicating anti-bunching (bunching). Here (Fig. 2), we probe the influence of the collective dynamics by varying the optical coupling strength  $J$ , while keeping the mechanical coupling  $K$  zero for clarity. We note that, when increasing  $J$ , the optical resonance effectively shifts:  $\Delta \rightarrow \Delta + J$ . To keep the photon number fixed while increasing  $J$ , the detuning has to be adapted [30]. In this setting, we observe that the interaction between the cells suppresses anti-bunching (inset of Fig. 2). Photon blockade is lost if the intercellular coupling becomes larger than the effective nonlinearity,  $2J \gtrsim g_0^2/\Omega$ . Above this value, the photon statistics shows bunching, and ultimately reaches Poissonian statistics for

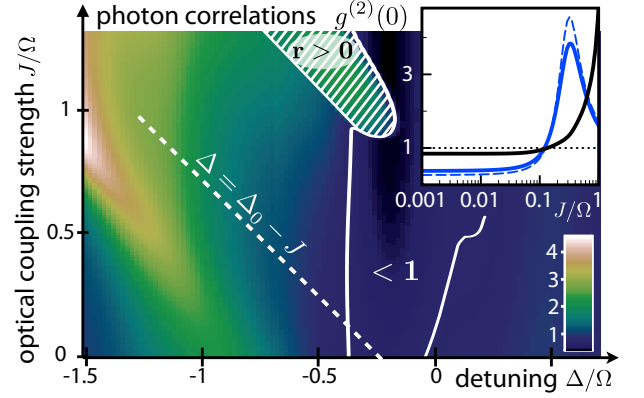


Figure 2. Loss of photon blockade for increasing optical coupling in an array of optomechanical cavities. The equal time photon correlation function shows anti-bunching ( $g^{(2)}(0) < 1$ ) and bunching ( $g^{(2)}(0) > 1$ ) as a function of detuning  $\Delta$  and optical coupling strength  $J$ . The smallest values of  $g^{(2)}(0)$  are found for a detuning  $\Delta_0 = -g_0^2/\Omega$ . When increasing the coupling  $J$  while keeping the intracavity photon number constant, i.e. along the dashed line, photon blockade is lost (inset,  $g^{(2)}(0)$  as black solid line). For a smaller driving power (inset, blue solid line,  $\alpha_L = 5 \cdot 10^{-5} \kappa$ ), anti-bunching is more pronounced, and the behavior is comparable to that of a nonlinear cavity (inset, dashed line). The hatched area in the main figure outlines a region where a transition towards coherent mechanical oscillations has set in (see main text and further figures).  $\kappa = 0.3 \Omega$ ,  $\alpha_L = 0.65 \kappa$ ,  $g_0 = 0.5 \Omega$ ,  $\Gamma = 0.074 \Omega$ .

large couplings. Similar physics has recently been analyzed for coupled qubit-cavity arrays, [30]. For very large coupling strengths, though, the density plot of Fig. 2 reveals signs of the collective mechanical motion (hatched area). There we observe the correlation function to oscillate (at the mechanical frequency) and to show strong bunching. We will now investigate this effect.

*Collective mechanical quantum effects.* - To describe the collective mechanical motion of the array, we focus on the case of purely mechanical intercellular coupling ( $K > 0$ ,  $J = 0$ ) for simplicity. Note, though, that the effect is also observable for optically coupled arrays, as discussed above.

As our main result, Figs. 3(a) and 4(a) show the sharp transition between incoherent self-oscillations and a phase-coherent collective mechanical state as a function of both laser detuning  $\Delta$  and coupling strength  $K$ : In the regime of self-induced oscillations, the phonon number  $\langle \hat{b}^\dagger \hat{b} \rangle$  reaches a finite value. Yet, the expectation value  $\langle \hat{b} \rangle$  remains small and constant in time. When increasing the intercellular coupling, though,  $\langle \hat{b} \rangle$  suddenly starts oscillating:

$$\langle \hat{b} \rangle(t) = \bar{b} + r e^{-i\Omega_{\text{eff}} t}. \quad (4)$$

Our more detailed analysis (see below) indicates that

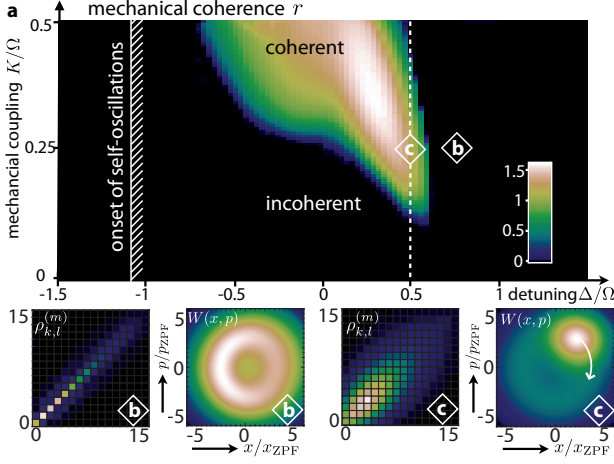


Figure 3. Transition from the incoherent to the synchronized (coherent) phase: (a) Mechanical coherence  $r$ , measuring the amplitude of the order parameter in  $\langle \hat{b} \rangle(t) = \bar{b} + r e^{-i\Omega_{\text{eff}} t}$ , as a function of laser detuning  $\Delta$  and mechanical coupling  $K$ . At weak coupling, the self-oscillations are incoherent,  $r = 0$ , due to quantum noise. When increasing the coupling strength, the systems shows a sharp transition towards the ordered regime, where the mechanical oscillations are phase-coherent,  $r > 0$ . A cut for fixed detuning (dashed line) is shown in Fig. 4(a). (b,c) Modulus of the density matrix elements (in Fock space) and Wigner density of the collective mechanical state in the incoherent (b) and the coherent regime (c) (as marked in (a)).  $g_0 = \kappa = 0.3\Omega$ ,  $\alpha_L = 1.1\kappa$ ,  $\Gamma = 0.074\Omega$

this transition results from the competition between the fundamental quantum noise of the system and the tendency of phase locking between the coupled nonlinear oscillators. Below threshold, the quantum noise from the phonon bath and the optical fields diffuses the mechanical phases at different sites and drives the mechanical motion into an incoherent mixed state. The reduced density matrix  $\hat{\rho}^{(m)}$  is predominantly occupied on the diagonal, see Fig. 3(d), and the Wigner distribution,  $W(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} \langle x-y | \hat{\rho}^{(m)} | x+y \rangle e^{2ipy/\hbar} dy$ , has a ring-like shape, reflecting the fact that the phase of the motion is undetermined [12, 31], see Fig. 3(b). Above threshold, the mechanical motion at different sites becomes phase locked, and the coherence parameter  $r$  (Eq. (4)) reaches a finite value. Within our mean-field ansatz,  $r$  grows with a critical exponent  $1/2$  close to the threshold, see Fig. 4(a). The emergence of coherence also becomes apparent from the off-diagonal elements of  $\hat{\rho}^{(m)}$  (Fig. 3(e)). The corresponding Wigner function assumes the shape of a coherent state with a definite phase oscillating in phase space, see Fig. 3(c). Thus, this transition spontaneously breaks the time-translation symmetry. In a two-dimensional implementation, true long range order is excluded, but the coherence between different sites is expected to decay as a power law with distance. We also note that this transition is the quantum mechanical analogon of classical synchronization, which was studied

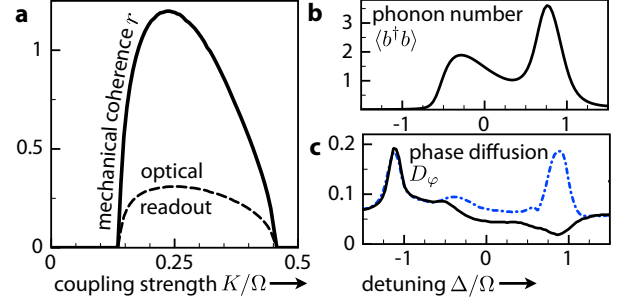


Figure 4. (a) The mechanical coherence  $r$  as a function of coupling strength  $K$  for fixed detuning  $\Delta = 0.5\Omega$ . The dashed line shows the optical readout of coherence, i.e. the oscillating component of the photon number  $\langle \hat{a}^\dagger \hat{a} \rangle$ , which is directly accessible in experiment. The loss of synchronization for even larger coupling strengths can be attributed to enhanced phase fluctuations and a nonmonotonic behavior of the effective phase coupling rate between the phases, as discussed in the main text. (b) The phonon number  $\langle \hat{b}^\dagger \hat{b} \rangle$  shows maxima at the resonance and at the sideband ( $\Delta - \Delta_0 \approx \Omega$ ). (c) The diffusion constant for the mechanical phase,  $D_\varphi$ , for an uncoupled ( $K = 0$ , solid line) and coupled array ( $K = 0.1\Omega$ , dash-dotted line). Other parameters as in Fig. 3.

for optomechanical systems in [32, 33]. An important difference is, though, that the classical nonlinear dynamics were analyzed for an inhomogeneous (with disordered mechanical frequencies) system in the absence of noise [32, 33], while in our case disorder is only introduced via fundamental quantum noise.

The laser detuning determines both the strength of the self-oscillations and the influence of the cavity shot noise on the mechanical motion. As we will show below, even the coherent coupling between the mechanical phases (ultimately leading to synchronization) is tunable via the laser frequency. As a result, the synchronization threshold depends non-trivially on the detuning parameter  $\Delta$ , see Fig. 3(a). The onset of self-induced oscillations happens continuously and already at a red-detuned laser frequency,  $\Delta < 0$ , due to the effect of the static mechanical shift and the shift of the optical spectrum by  $\Delta_0 = -g_0^2/\Omega$ . Close to the onset, the mechanical phases are very susceptible to quantum noise preventing synchronization. The diffusion rate  $D_\varphi$ , defined as the linewidth of the correlator  $\langle \hat{b}(t) \hat{b}^\dagger(0) \rangle \sim e^{-(i\Omega_{\text{eff}} + D_\varphi)t}$ , shows a maximum (Fig. 4(c), solid line). For finite coupling strengths, the diffusion is enhanced, most strikingly at the mechanical sideband. As a result, the synchronization threshold shows a minimum between the onset of self-oscillations and the sideband. From extended simulations, we find that this behavior is generic for systems in the resolved sideband regime ( $\Omega > \kappa$ ).

*Semi-classical description.* - In order to gain further insight into the coupling and decoherence mechanisms, we analyze a semi-classical model for coupled Hopf oscil-

lators with phases  $\varphi_j$  and amplitudes  $A_j$  (in units of the mechanical ground state width). We follow [32], but add noise terms:

$$\begin{aligned}\dot{\varphi}_j &= -\Omega - \frac{2g_0}{A_j} |\alpha|^2 \cos \varphi_j + \frac{K}{zA_j} \sum_{\langle ij \rangle} A_i \cos \varphi_i \cos \varphi_j + \frac{\tilde{\xi}_\varphi}{A_j} \\ \dot{A}_j &= \gamma(A_j - \bar{A}) + \frac{K}{zA_j} \sum_{\langle ij \rangle} A_i \cos \varphi_i \sin \varphi_j + \tilde{\xi}_A.\end{aligned}\quad (5)$$

The radiation pressure forces the mechanical motion onto a limit cycle with steady state amplitude  $\bar{A}$  and amplitude decay rate  $\gamma$ , and also modifies the mechanical oscillation frequency via an amplitude dependent optical spring effect:  $\Omega(A) = \Omega - 2g_0 \langle |\alpha|^2 \cos \varphi \rangle_T / A$ , where  $\langle \dots \rangle_T$  denotes the average over one mechanical period. The fluctuating noise forces  $\tilde{\xi}_\varphi$  and  $\tilde{\xi}_A$  comprise the effects of the phonon bath and cavity shot noise [34]. The Hopf equations (5) can be derived directly from the Langevin equations of the optomechanical system, and a description of this kind has proven good qualitative agreement with the full quantum dynamics for a single optomechanical cell [12, 34]. The complexity of equations (5) can be reduced by integrating out the amplitude dynamics [32]. In addition, we apply a mean-field approximation: We replace  $e^{i\varphi_j}$  for neighboring cells by  $\langle e^{i\varphi_j} \rangle \equiv R e^{i\Psi}$  and  $e^{i2\varphi_j}$  by  $\langle e^{i2\varphi_j} \rangle \equiv R_2 e^{i\Psi_2}$ , where  $\langle \dots \rangle$  denotes the average over all sites [35]. The resulting equation describes the coupling of the mechanical phase on a single site,  $\varphi$ , to a mean field  $\Psi$ :

$$\begin{aligned}\dot{\varphi} &= -\Omega(\bar{A}) + KR \cos(\Psi - \varphi) + K_1 R \sin(\Psi - \varphi) \\ &\quad + K_2 R^2 \sin(2\Psi - 2\varphi) + K_2 R R_2 \sin(\Psi_2 - \Psi - \varphi) + \xi_\varphi,\end{aligned}\quad (6)$$

where we defined the coupling rates  $K_2 = K^2/2\gamma$  and  $K_1 = K d\Omega/\gamma - K_2$  and a fluctuating noise force  $\xi_\varphi$ , which is characterized by a diffusion constant  $D_\varphi$  [34], to be discussed below. The differential  $d\Omega = \bar{A} \frac{d\Omega}{dA}|_{A=\bar{A}}$  accounts for the variation of the mechanical frequency due to fluctuations in the amplitude. It turns out to be decisive, since it determines the strength and the sign of the (most important) coupling rate  $K_1$ .

The phase equation (6) has the form of the mean-field Kuramoto equation [35] in the presence of noise, with additional coupling terms proportional to  $K$  and  $K_2$ . In the incoherent regime, the order parameter  $R$  ( $R_2$ ) is zero and the phase fluctuates freely. In the coherent regime, i.e. for  $0 < R$ ,  $R_2 < 1$ , on the other hand, the different coupling terms force the phase  $\varphi$  towards a fixed phase relation with  $\Psi$ . Note that within the mean-field approximation, only a configuration with  $\varphi = \Psi$  is stable. This state can be achieved via the coupling terms proportional to  $K_1$  and  $K_2$ , which provide a restoring force  $\sim K_{1,2}R$

acting on  $\varphi$ . The cosine term only renormalizes the oscillation frequency. This statement can be clarified by a linear stability analysis following the approach of Strogatz *et al.* [36] for the generic Kuramoto model. At the threshold of synchronization, the time evolution of the small increment  $\delta R$  on top of the incoherent background ( $R = 0$ ) is governed by  $\delta \dot{R} = (K_1 - 2D_\varphi)\delta R + \mathcal{O}(\delta R^2)$ , where the coupling terms  $K$  and  $K_2$  enter only as higher order contributions. As a result, the incoherent phase becomes unstable for

$$K_1 = 2D_\varphi, \quad (7)$$

defining the threshold of the transition. Moreover, if  $K_1$  becomes negative, no stable phase synchronization is possible. This situation arises if  $d\Omega < 0$ , or for large intercellular coupling rates  $K > 2d\Omega$ , as observed in Fig. 4(a).

The total phase diffusion rate is given for vanishing coupling ( $K = 0$ ) by  $D_\varphi = \frac{1}{A^2}(\tilde{D}_\varphi + \frac{\delta\Omega^2}{\gamma^2}\tilde{D}_A)$  [34], where the diffusion rates  $\tilde{D}_{\varphi,A}$  correspond to the noise terms  $\tilde{\xi}_{\varphi,A}(t)$  of Eq. (5) and are defined by  $2\tilde{D}_{\varphi,A} = \int d\tau \langle \tilde{\xi}_{\varphi,A}(t + \tau) \tilde{\xi}_{\varphi,A}(t) \rangle_T$ . For the explicit expressions we refer to [34], and discuss the main qualitative feature here: Close to the threshold of self-oscillations, i.e. for small amplitudes and small amplitude damping  $\gamma$ , the phase is highly susceptible to noise. Moreover, while a large frequency differential  $\delta\Omega$  and a small value of  $\gamma$  increase the strength of the effective coupling rate  $K_1$ , this effect is overwhelmed by the quadratically enhanced diffusion rate. Thus, the semiclassical analysis affirms our observations of the quantum dynamics of the system. We note, though, that the semiclassical equations neglect higher-order correlations and can not fully capture the quantum dynamics in the regime of strong coupling,  $g_0 \gtrsim \kappa$ .

*Experimental prospects.* - We note that observation of the mechanical phase transition does *not* require single photon strong coupling ( $g_0 \gtrsim \kappa$ ): The quantum fluctuations of the light field will dominate over thermal fluctuations as long as  $g_0^2 |\alpha|^2 / \kappa > k_B T / Q$ . This is essentially the condition for ground-state cooling, which has been achieved utilizing high- $Q$  mechanical resonators and cryogenic cooling [1, 2]. In contrast, we note that the photon-blockade effect (Fig. 2) requires low temperatures  $T$  and  $g_0^2 \gtrsim \Omega \kappa$ , or at least, in a slightly modified setup [37, 38],  $g_0 \gtrsim \kappa$ . While still challenging, optomechanical systems are approaching this regime [39].

The flexibility to design two dimensional arrays has been demonstrated for optomechanical crystals in [40], and sufficiently strong optical and mechanical hopping rates are feasible, as shown for comparable structures in [32]. The strongest challenge would likely be the simultaneous optical driving of many cells, although similar physics may be observed for many cells coupling to one extended optical mode [5, 32, 33] (thereby effectively realizing global coupling). We expect the transition to

be robust against disorder [32]. One could also study the formation of vortices and other topological defects induced by engineered irregularities and periodic variations, and explore various different lattice structures. Thus, optomechanical arrays provide a novel, integrated and tunable platform for studies of quantum many body effects.

The authors would like to thank Oskar Painter and Björn Kubala for valuable discussions. This work was supported by the DFG Emmy-Noether program, an ERC starting grant, the DARPA/MTO ORCHID program and the ITN network cQOM.

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